Arithmetic Sequences, Part 1

SEQUENCES

In an **arithmetic sequence**, the same amount is always added to get from one term to the next.

The amount that is added to get to each next term is called the **common difference**.
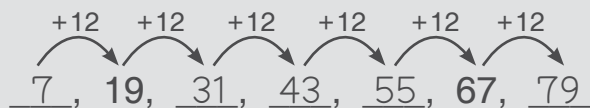
EXAMPLE | Fill in the blanks to complete the arithmetic sequence below.

___, 19, ___, ___, ___, 67, ___

Arithmetic sequences are really just skip-counting patterns!

We begin by finding the common difference. To get from 19 to 67 in this sequence, we add the common difference 4 times.

___, 19, ___, ___, ___, 67, ___

Adding the common difference 4 times adds a total of $67 - 19 = 48$. So, the common difference is $48 \div 4 = 12$. We use this to find the missing terms, as shown.

$+12 \quad +12 \quad +12 \quad +12 \quad +12 \quad +12$

_7_, 19, _31_, _43_, _55_, 67, _79_

PRACTICE | Find the common difference for each arithmetic sequence below.

35. 7, 16, 25, 34, 43, ...

36. -33, -25, -17, -9, -1, ...

35. _____

36. _____

37. 29, 26, 23, 20, 17, ...

38. __, 21, __, 35, __, ...

37. _____

38. _____

39. 74, __, __, 41, __, ...

40. 30, __, __, __, __, $32\frac{1}{2}$, ...

39. _____

40. _____

PRACTICE | Fill in the blanks to complete each arithmetic sequence below.

41. 19, _____, _____, 64, _____, _____, 109

42. 98, _____, 112, _____, _____, _____, 140

43. _____, _____, _____, _____, 32, $30\frac{1}{2}$, _____

44. 10, _____, _____, _____, _____, 22, _____

PRACTICE | Answer each question below.

45. What is the common difference of an arithmetic sequence whose first term is 25 and whose tenth term is 115?

45. _____

46. What is the common difference of an arithmetic sequence whose 23rd term is $\frac{1}{3}$ and whose 25th term is $\frac{1}{2}$?

46. _____

47. An arithmetic sequence has 10th term 4 and 30th term 68. What is the 20th term of the sequence?

47. _____

48. ★ An arithmetic sequence has five terms. The first term is 40, and the sum of all five terms is 80. What is the common difference?

48. _____

SEQUENCES

Arithmetic Sequences, Part 1

EXAMPLE | What is the 50th term of the arithmetic sequence below?

$$2, \ 5, \ 8, \ 11, \ 14, \ 17, \ ...$$

The first term of the arithmetic sequence is 2 and the common difference is 3.

$$\overset{+3 \quad +3 \quad +3 \quad +3 \quad +3}{2, \ \ 5, \ \ 8, \ \ 11, \ 14, \ 17, \ ...}$$

To get to the 2nd term, we add 1 three to 2.
To get to the 3rd term, we add 2 threes to 2.
To get to the 4th term, we add 3 threes to 2.

To get to the 50th term, we add 49 threes to 2.
So, the 50th term is $2+49(3) = 2+147 = \textbf{149}$.

PRACTICE | Find the value of the missing term listed for each arithmetic sequence below.

49. 15, 19, 23, 27, 31, ..., ___
$\underset{10th}{}$

50. -11, -6, -1, 4, 9, ..., ___
$\underset{40th}{}$

51. 5, -2, -9, -16, -23, ..., ___
$\underset{15th}{}$

52. -29, -19, -9, 1, 11, ..., ___
$\underset{100th}{}$

PRACTICE | Answer each question below.

53. What is the 13th term of an arithmetic sequence whose first term is 9 and whose common difference is 8?

53. _____

54. What is the first term of an arithmetic sequence whose 100th term is 40 and whose common difference is $\frac{1}{3}$?

54. _____

55. The 12th and 15th terms of an arithmetic sequence are 85 and 106. What is the first term of the sequence?

55. _____

EXAMPLE | Write an expression for the nth term of the arithmetic sequence below.

$$-1, \ 4, \ 9, \ 14, \ 19, \ 24, \ ...$$

The first term of the sequence is -1 and the common difference is 5.

$$\overset{+5 \ \ +5 \ \ +5 \ \ +5 \ \ +5}{-1, \ \ 4, \ \ 9, \ \ 14, \ \ 19, \ \ 24, \ ...}$$

To get the 2nd term, we add 1 five to -1.
To get the 3rd term, we add 2 fives to -1.
To get the 4th term, we add 3 fives to -1.

To get the nth term, we add $(n-1)$ fives to -1.

So, the nth term is $-1+(n-1)5$. Distributing the 5 and simplifying gives

$$-1+(n-1)5 = -1+5n-5$$
$$= \mathbf{5n-6}.$$

> You can check your answer by plugging in values for n!
> For example, we know the 3rd term is 9. So, we can plug in $n=3$ to get $5(3)-6=9.$ ✓

PRACTICE | Write a simplified expression for the nth term of each arithmetic sequence below.

56. 18, 24, 30, 36, ..., _____
    n^{th}

57. 4, 19, 34, 49, ..., _____
    n^{th}

58. -13, -5, 3, ..., _____
    n^{th}

59. $\frac{9}{4}, \frac{5}{2}, \frac{11}{4}, ...,$ _____
    n^{th}

PRACTICE | Answer each question below. Simplify all expressions.

60. An arithmetic sequence has first term a and common difference 3. Write an expression for the 20th term of the sequence.

60. _____

61. An arithmetic sequence has first term 6 and common difference d. Write an expression for the 101st term of the sequence.

61. _____

62. ★ In an arithmetic sequence, the 1st term is 20, the 2nd term is 32, and the kth term is 500. What is k?

62. $k =$ _____