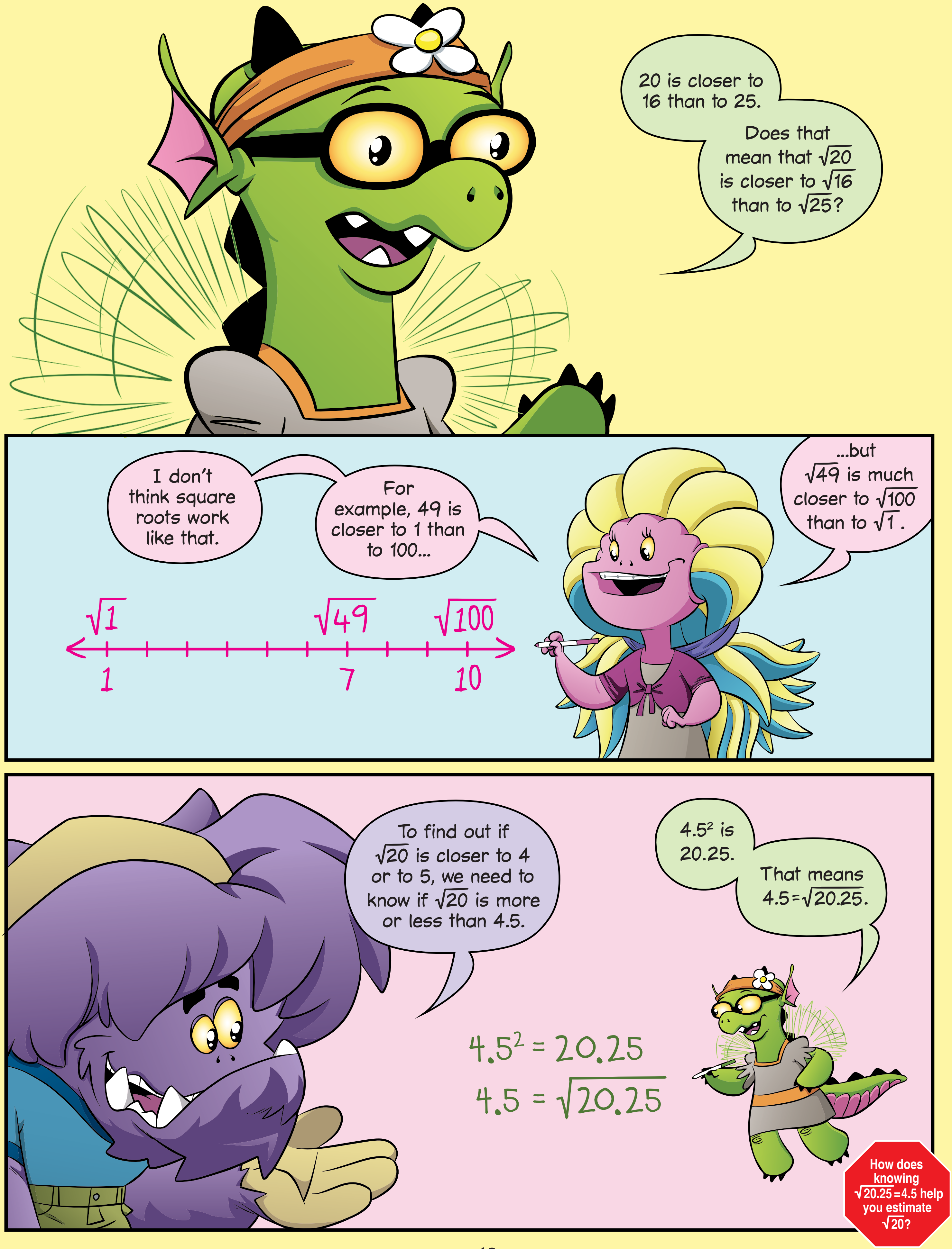
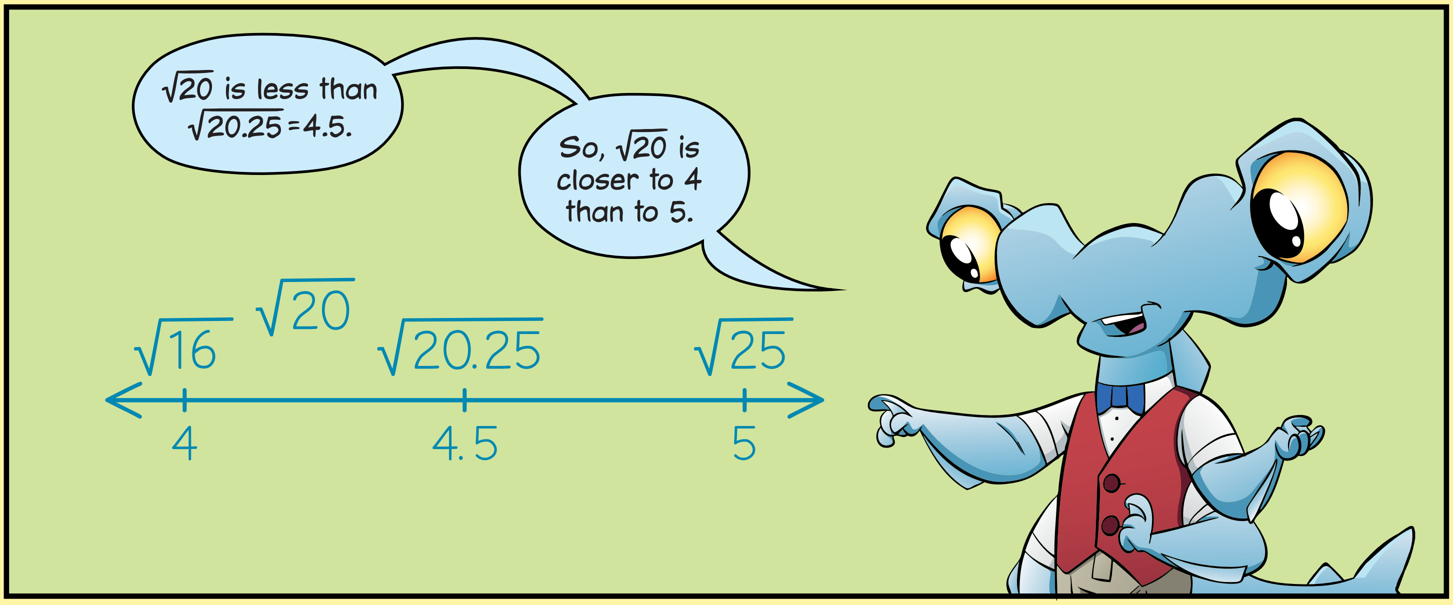
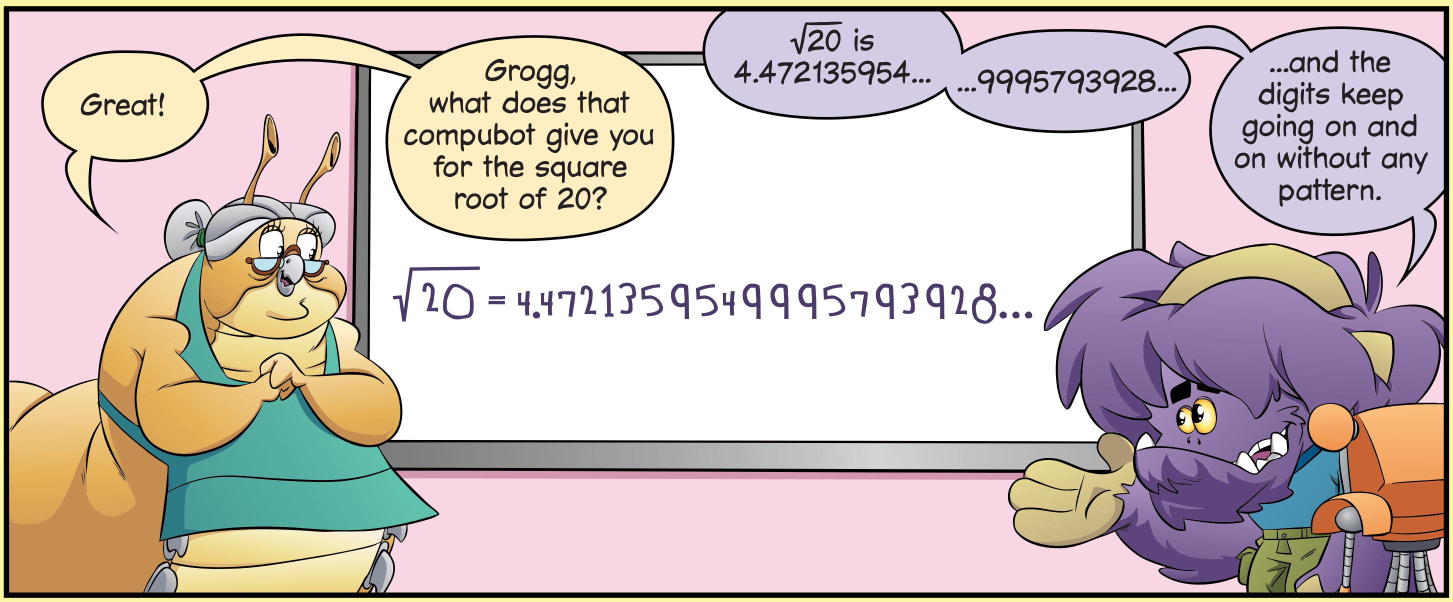
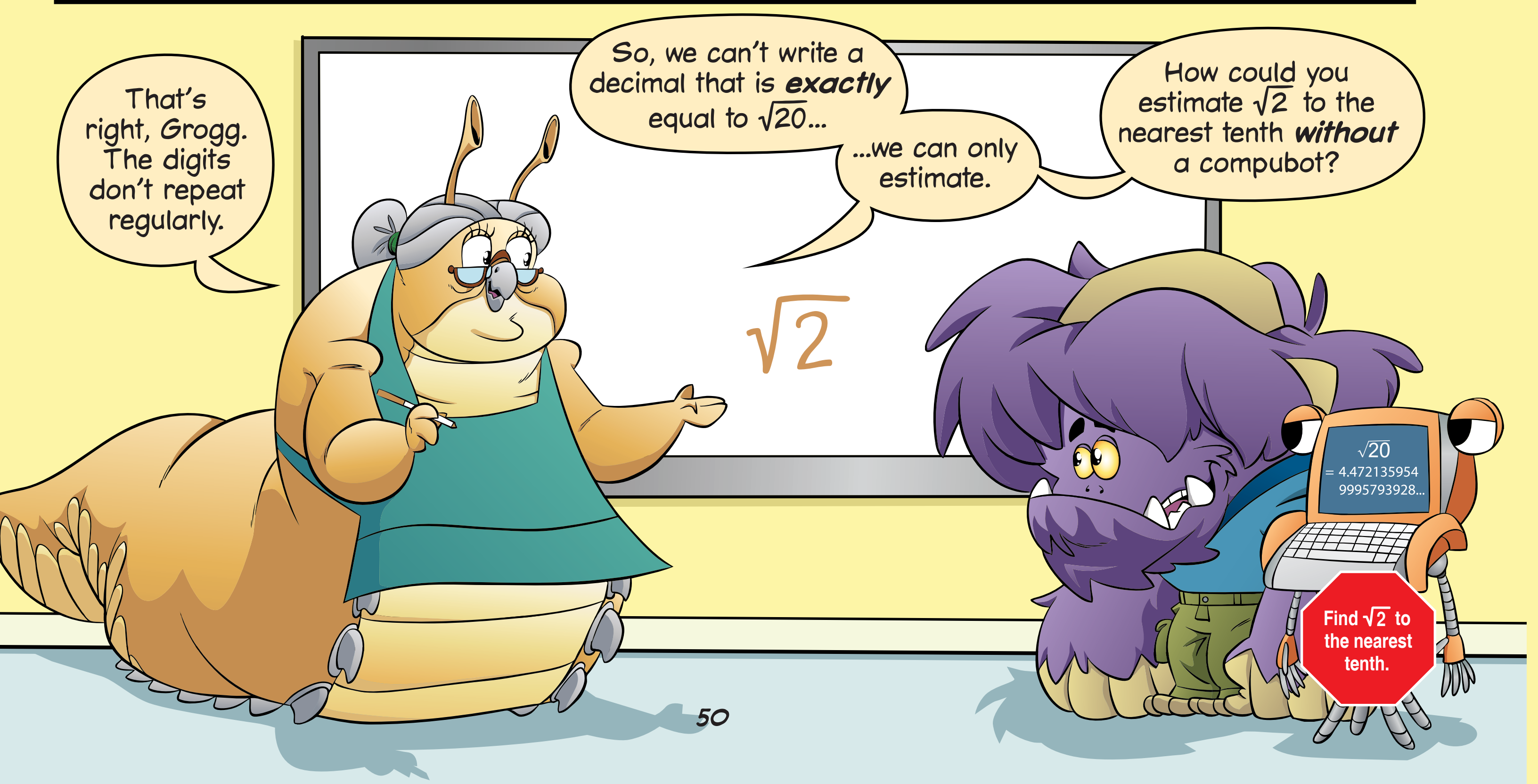
Ms. Q. Estimation
Ms. Q.?
I know the square roots of perfect squares like 25, 81, and 144.
Yes, Grogg?
But, how do I find the square root of other numbers, like 20?
Does 20 even have a square root?
$\sqrt{25} = 5$
$\sqrt{81} = 9$
$\sqrt{144} = 12$
$\sqrt{20} = ?$
The square root of 20 is between 4 and 5.
How can you tell?
Because $\sqrt{20}$ is between $\sqrt{16}$ and $\sqrt{25}$.
I see. $\sqrt{20}$ is more than $\sqrt{16} = 4$...
...but less than $\sqrt{25} = 5$.
That's right. Is $\sqrt{20}$ closer to 4 or to 5?
$\sqrt{16}$
$\sqrt{20}$
$\sqrt{25}$
4
5
FOR ANY TWO NONNEGATIVE NUMBERS a AND b, IF $a < b$, THEN $\sqrt{a} < \sqrt{b}$. SO, $\sqrt{16} < \sqrt{20} < \sqrt{25}$.


20 is closer to 16 than to 25.
Does that mean that $\sqrt{20}$ is closer to $\sqrt{16}$ than to $\sqrt{25}$?
I don't think square roots work like that.
For example, 49 is closer to 1 than to 100...
...but $\sqrt{49}$ is much closer to $\sqrt{100}$ than to $\sqrt{1}$.
$\sqrt{1}$ $\sqrt{49}$ $\sqrt{100}$
1 7 10
To find out if $\sqrt{20}$ is closer to 4 or to 5, we need to know if $\sqrt{20}$ is more or less than 4.5.
4.5^2 is 20.25.
That means $4.5=\sqrt{20.25}$.
$4.5^2 = 20.25$
$4.5 = \sqrt{20.25}$
How does knowing $\sqrt{20.25}=4.5$ help you estimate $\sqrt{20}$?


√20 is less than √20.25 = 4.5.
So, √20 is closer to 4 than to 5.
√16
√20
√20.25
√25
4
4.5
5


Great!
Grogg, what does that compubot give you for the square root of 20?
√20 is 4.472135954...
...9995793928...
...and the digits keep going on and on without any pattern.
√20 = 4.4721359549995793928...


That's right, Grogg. The digits don't repeat regularly.
So, we can't write a decimal that is **exactly** equal to √20...
...we can only estimate.
How could you estimate √2 to the nearest tenth **without** a compubot?
√2
√20 = 4.472135954 9995793928...
Find √2 to the nearest tenth.